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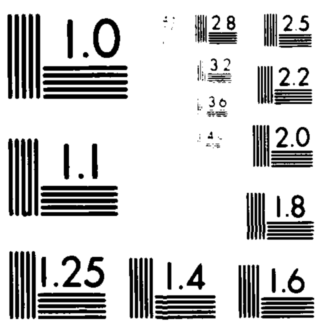

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IMPACT AND PENETRATION PROBLEMS

FINAL REPORT

A.C. Eringen and N. Ari

March 1981

U.S. Army Research Office
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Princeton University

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Nonlocal elasticity, nonlocal plasticity, fracture criteria, punch and penetration problems

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

Impact and Penetration phenomena are investigated by recently established nonlocal continuum theories. Constitutive equations are developed for nonlocal elastic and plastic materials. The problems of nonlocal line crack, half space under punch, plug formation and perforation of visco-plastic plates are solved. The absence of singularities made it possible to introduce several fracture and penetration initiation criteria based on maximum stress hypotheses.

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I. INTRODUCTION

High speed penetration mechancs involves highly complicated physical phenomena, all operating more or less simultaneously, depending on the speed of the projectile and the nature of the target material. Classical theories of elasticity, plasticity and visco-elasticity are often inadequate, especially in quantitative terms, in explaining these penetration problems. Even at moderate speeds, fracture,plastic flow and new surface initiation occurs. Classical discipines are not designed to deal with these types of physical phenomena.

In order to obtain a basic understanding of penetration phenomena, the isolation of fundamental failure mechanisms and their rigorous mathematical treatment are necessary. Moreover, in order to avoid ad-hoc assumptions, the theoretical underpinnings of such an approach should accommodate the effects of microstructure and long range interactions.

To achieve the above goals, the nonlocal continuum theories have been developed and have been subjected to acid tests of model problems.

The results have been most gratifying. After much laborious efforts to solve complicated integro-differential equations for mixed boundary-value problems, it has been possible to establish simple, meaningful fracture criteria. Cricial stresses and/or velocities are obtained for the initiation of penetration and fracture. The estimates for the cohesive stresses of various materials have been in the right range, i.e. those accepted by solid state physicists and by metalurgists.

On the theoretical side, further research has been devoted toward improving nonlocal theories and fine tuning them for problem-solving purposes. The correspondence between the nonlocal theories and solid state physics has been further delineated. Finally, a rigorous thermodynamic derivation of nonlocal plasticity has been given.

Below, first we present certain selected research results. After brief descriptions of the solved problems, the important conclusions are restated. In the next section, the new theoretical work in nonlocal formulation is summarized.

The emerging pattern of the interplay of the theoretical developments and the solutions of the model problems is rather encouraging, in that it enables us to formulate a unified failure criteria. Further intensified research is necessary to extend these basic results to cases where the physics of the penetration phenomena are considered in more detail.

II. RESEARCH ACCOMPLISHED

1. *Nonlocal Fracture Criterion*

A plate with line crack, subject to uniform tension at infinity perpendicular to the direction of line crack is a crucial problem for the understanding of the rupture of brittle materials. The classical solution of the Griffith problem predicts an infinite hoop stress at the tip of the crack. Because of this singularity, a perfectly adequate criterion of brittle fracture -- the maximum stress hypothesis -- used for all other types of bodies with no sharp geometric changes, had to be abandoned. To this end Griffith [1] introduced his celebrated fracture criterion which he deduced by equating the elastic energy to "the surface tension energy"

$$(1) \qquad t_0^2 \ell = C_G \qquad C_G = \frac{2E}{(1-\gamma^2)} \gamma$$

where t_0 is the applied tension, ℓ is the half crack length. The Griffith constant C_G is expressed in terms of Young's modulus E, Poisson's ratio ν, and the surface tension energy γ.

The Griffith criterion depends on the surface energy concept which itself is not included in the classical elasticity theory. In order to derive a simple, consistent fracture criterion, a better description of the crack tip stresses is necessary.

Eringen, et al [2] solved the crack tip problem by means of nonlocal elasticity. They deduced the following significant results

i) The stress at the crack tip is finite [Fig. 1]. It is given by

$$t_{yy}(\ell,0) = C(2\ell/a)^{\frac{1}{2}}\, t_0 \qquad C \simeq 0.73 \tag{2}$$

where t_0 is the applied uniform tension at infinity.

ii) When the maximum stress is equal to the cohesive stress t_c, the fracture will occur. In this case, the nonlocal fracture criterion is given by

$$t_0^2\,\ell = (a/2C^2)t_c^2 = C_G \tag{3}$$

where a is the atomic distance. Surprisingly enough, this is the Griffith criterion with the added benefit that the Griffith constant is now determined theoretically.

iii) By utilizing the formal similarity between the two criteria (1) and (3), we can predict the theoretical cohesive strengths of various crystals. Once the experimental value for γ is given, the calculations can be carried out easily to determine the cohesive stresses. In Table 1, the nonlocal estimates are compared to those of atomistic models [3]. The remarkably close values obtained are once again indicative of the far reaching power of nonlocal theories.

2. *Initial Penetration by a Rigid Punch*

One of the most fundamental problems of penetration mechanics is that of an elastic half space under an indenting punch [Fig. 2]. The classical solution to this contact problem [4] contains stress singularities at the edges of the penetrator no matter how small the applied loads may be. Hence it is very difficult to predict the critical stress for the initiation of penetration. The classical elasticity fails to apply in the small region around the edges of the indenting block. And it is exactly this region which is crucial to our understanding of the penetration phenomena.

The state of the stress around the edges is heavily influenced by the microstructure and by the long range interatomic interactions. The recently established nonlocal theories incorporate these effects naturally [5]. To this end, the rigid punch problem has been solved by means of the nonlocal elasticity theory. The stress and displacement fields are obtained by solving a set of complicated dual integral equations [6].

The variation of nonlocal stress at the surface is plotted in Fig. 3. The maximum stress is given by

$$(4) \qquad t_{max}/t_0 = 0.25\sqrt{\varepsilon} \qquad \varepsilon = a/2k\ell$$

where t_0 is the applied average stress, k is an attenuation factor having a value in the neighborhood of unity, a is the atomic distance and 2ℓ is the punch width.

The nonlocal analysis of the rigid punch problem leads to the following conclusions:

i) Nonlocal elasticity predicts finite stresses at the points of contact of the half space with the edges of the penetrator.

ii) By equating the maximum stress to the cohesive stress, t_c

$$p_{cr}^2 \ell = 8 t_c^2 a/k \qquad (k \simeq 1) \tag{5}$$

it has been possible to derive a penetration initiation criterion for static loads and brittle solids. Note that in arriving at this result, no free parameter adjustment has been used.

3. *Plug Formation*

The impact failure of a target plate hit by a projectile occurs by the simultaneous action of various failure mechanisms. One of these basic mechanisms is that of plug formation (for an extensive review of impact phenomena, see e.g. Backman and Goldsmith [7]). The main line of approach to high speed penetration problems has been the so-called hydrodynamic model which employs the classical viscous fluid theory in various modified and simplified forms. The inadequacy of classical continuum theories in describing the high gradient phenomena in rupture regions has already been discussed.

Thus a nonlocal viscoelasticity theory has been introduced to account for the high strain rate effects in plug formation [8]. The effect of the impact is represented by a uniform initial velocity representation over a circular region of the plate surface. Since the plate is very thin, only the vertical shearing stresses are considered. The analytical solution to the mixed boundary value problem yields the nonlocal displacement and stress fields. The nonlocal parameter ε is determined by matching the initial nonlocal displacement value at the center of the plug $(t = 0,\ r = 0)$ to the experimental result given by Pytel and Davids [9]. The results are plotted in Figures 4 and 5.

The time decay and space distribution of the shear stress can be used in design considerations. The viscous effects cause high stresses and small displacements in the first few microseconds after impact. As the time increases, the maximum stress decreases rapidly and the displacement continues to increase at a slower rate.

The present solution has the following important characteristics:

(i) In contrast to the classical analysis, the viscous stresses at the periphery of the plug are found to be finite.

(ii) The critical velocity for the initial plug formation is given by:

$$V_{cr} = (R/2.623\,\mu)\,t_c \tag{6}$$

where t_c is the cohesive shear strength, μ is the shear modulus and R is the projectile radius. Note that this result cannot be arrived at by the classical theory, since the maximum stress is infinite and therefore cannot be used to deduce a non-zero impact velocity.

4. *Perforation of Visco-Plastic Plates*

The high speed penetration of a projectile into a plate requires the consideration of visco-plastic effects. In addition, the nonlocal intermolecular forces must be taken into account. In [10], a nonlocal analysis of plug formation in visco-plastic plates is given.

The model problem considers an infinite plate of thickness h_0 being hit by a cylindrical rigid projectile. [Fig. 6]. The incidence is normal and the initial velocity is V_0. It is supposed that after such an impact, a cylindrical portion of the plate with radius R is set into uniform motion. Since the plate is assumed to be sufficiently thin, only the effects of the anti-plane shear stress are considered.

The nonlocal analysis generalized the classical Bingham solids. Using appropriate nonlocal constitutive equations, which include the plastic yielding, calculations are carried out to obtain the stress and displacement fields. The important results can be summarized as

(i) The maximum value of the shear stress is finite and, when it reaches the cohesive value, the penetration begins. The maximum stress is given by

$$t_{rz} = -\sigma - \frac{2}{3}\exp(-2/\varepsilon)\, I_1(2/\varepsilon) \tag{7}$$

where

$$\varepsilon = a^2/k^2R^2 \tag{8}$$

and I_1 is the modified Bessel's function, R is the radius of the projectile, a is the atomic distance, $k \simeq 1$ is the attenuation parameter and σ is the ratio of the yield stress to the viscous stress $(\mu V_0/R)$, μ is the shear modulus.

(ii) Variations of plate thickness and velocity drop with perforation time and velocity drops with plate thickness are plotted in Figs. 7 and 8. Due to plastic effects, longer periods are required for complete perforation. The same is valid for the velocity drop. It is much larger when the plastic effects are considered.

5. *Nonlocal Elasticity and Lattice Dynamics*

The nonlocal theory of elasticity incorporates the effects of the discreteness of the lattice and of the long range atomic interactions. The similarity between the underlying physical and mathematical structures of the lattice dynamics and the nonlocal elasticity can be very useful. For example, the continuum formulation of nonlocal theories may obliviate the necessity of solving rather a large number of equations of motion of the discrete lattice.

The correspondence between the lattice dynamic and the nonlocal elasticity becomes especially important for materials with defect structures. In the case of real (imperfect) materials, the equations of the lattice dynamics become exceedingly complex, whereas the only change required in nonlocal field equations is in the form of nonlocal kernels.

In [11], passage is made from nonlocal elasticity to lattice dynamics. Equations of motion and total potential energy of both theories are compared and found to be identical. It is further demonstrated that:

(i) The nonlocal elastic moduli can be obtained by matching the dispersion relations of plane waves in nonlocal elastic solids to those of lattice dynamics.

(ii) The nonlocal elasticity theory intrinsically includes the surface effects and it can be naturally used to solve problems for imperfect finite elastic bodies.

6. *Continuum Mechanics at the Atomic Scale*

In [12] and [13], the difficulties in applying the classical continuum theories to problems, where physical or geometrical discontinuities exist, are delineated. The circumstances when the nonlocal theories can be most useful are discussed.

The thermodynamic restrictions and the rigorous development of nonlinear nonlocal elasticity is presented with special emphasis on the functional characteristics of the constitutive equations. From the considerations given there, the following conclusions can be derived:

(i) When the characteristic length of the external disturbance λ is of the same order of the internal characteristic length a (e.g. the lattice parameter), the classical continuum theories fail to apply. One of their basic premises $\lambda/a \gg 1$ is no longer true. In these instances, if one is not interested in the individual motions of atoms, nonlocal theories will yield sufficiently accurate results for the cooperative macroscopic phenomena of the real bodies (e.g. cracks, dislocations, etc.).

(ii) The functional character of nonlocal constitutive equations enables one to distinguish between the symmetry of the molecular orientations and the overall symmetry of the macroscopic body. This characteristic of nonlocal theories may serve as a powerful tool in cases where the local defect symmetry becomes very important (e.g., bifurcation of running cracks).

7. *Nonlocal Plasticity*

The nonlocal character of plastic deformations and of plastic yielding play an important role in impact and penetration problems. The long range interactions between the individual dislocations lead to the cooperative phenomenon of yielding. The regional loss of stability and the subsequent plastic flow clearly indicates the necessity of a rigorous thermodynamic treatment of plasticity.

In [14], a theory of nonlocal plasticity is developed in which the state of stress at a point of the body depends on elastic <u>and</u> plastic strains at <u>all</u> points of the body. A plastic strain measure is introduced which is not necessarily connected with the external strain measure. Thermodynamic treatment given in [14] generalizes the approach of Green and Naghdi and their coworkers [15-17] to include the nonlocal interactions. Using special types of functionals (additive functionals in the sense of Friedman and Katz [18]), constitutive equations are obtained for anisotropic and isotropic materials.

The systematic inclusion of nonlocal effects into plasticity theory provides the framework for the mathematical treatment of the complicated problems in plasticity and microplastic phenomena. In the theoretical formulation, the following results are worth noting:

(i) The yield criterion is derived from the second law of thermodynamics. The "strain-space" formulation (i.e., using the elastic and plastic strains as independent variables rather than the stress and plastic strains), provides a self-consistent set of constitutive response functionals.

(ii) Constitutive equations are obtained for the rate of plastic strain.

(iii) Special constitutive equations are given for the nonlocal isotropic elastoplastic solids, with no work hardening and with von Mises-type yield criterion.

III. TABLES AND FIGURES

TABLE I

Cohesive Stress. t_c: $t_c^2 a = K\gamma$; $K = 8C^2\mu/\pi(1-\nu)$; a = interionic dist.; γ = Surface energy.

		Experimental[1]					Present Theory				Atomistic Models
Type	Crystal	γ CGS	$\mu \times 10^{-11}$ CGS	ν	a A°	C	$K \times 10^{-11}$ CGS	$K\gamma$ $\times 10^{-14}$	t_c $\times 10^{-11}$	t_c/E	t_c/E
Face C.	Al	840	2.51	0.347	2.86	0.73	5.216	4.362	1.238	0.183	
Face C.	Ni	1725	7.48	0.276	2.49	0.73	14.020	24.185	3.117	0.163	
Body C.	Fe	1975	6.92	0.291	2.48	0.73	13.245	26.158	3.248	0.182	0.23
Ionic	LiF	480	4.40	0.068	2.014	0.73	6.407	3.075	1.236	0.131	
Diam.	C	5400	50.9	0.187	1.54	0.73	84.960	458.782	17.260	0.143	0.17
Hex.	Zn	575	3.83	0.333	2.66	0.73	7.792	4.481	1.298	0.127	0.11

Atomistic results are from Table 7.1, p. 160, Lawn and Wilshaw [3].

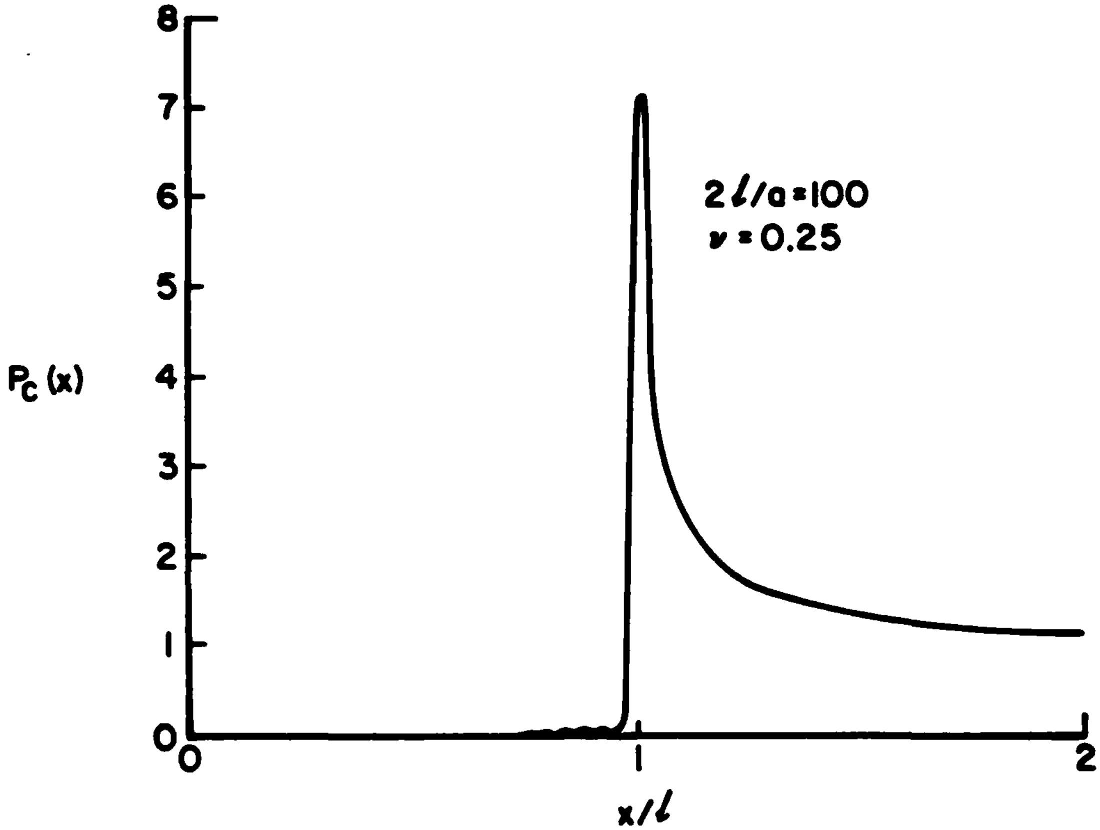

Figure 1: Stress Concentration Along the Crack Line [2]

$$P_c(x) = t_{yy}(x,0)/t_0 + 1$$

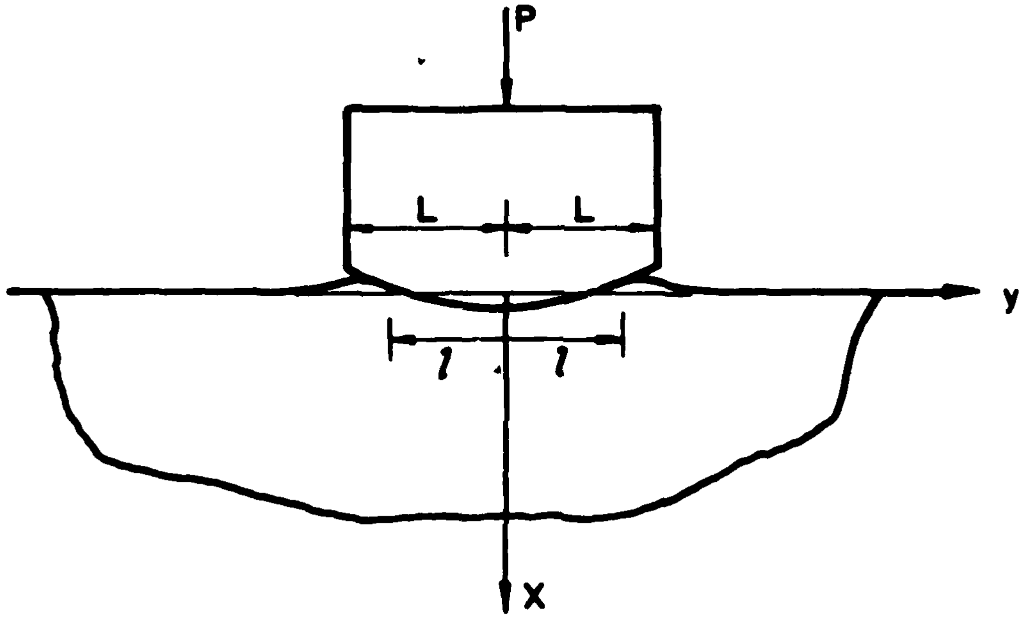

Figure 2: Half Space Under Punch

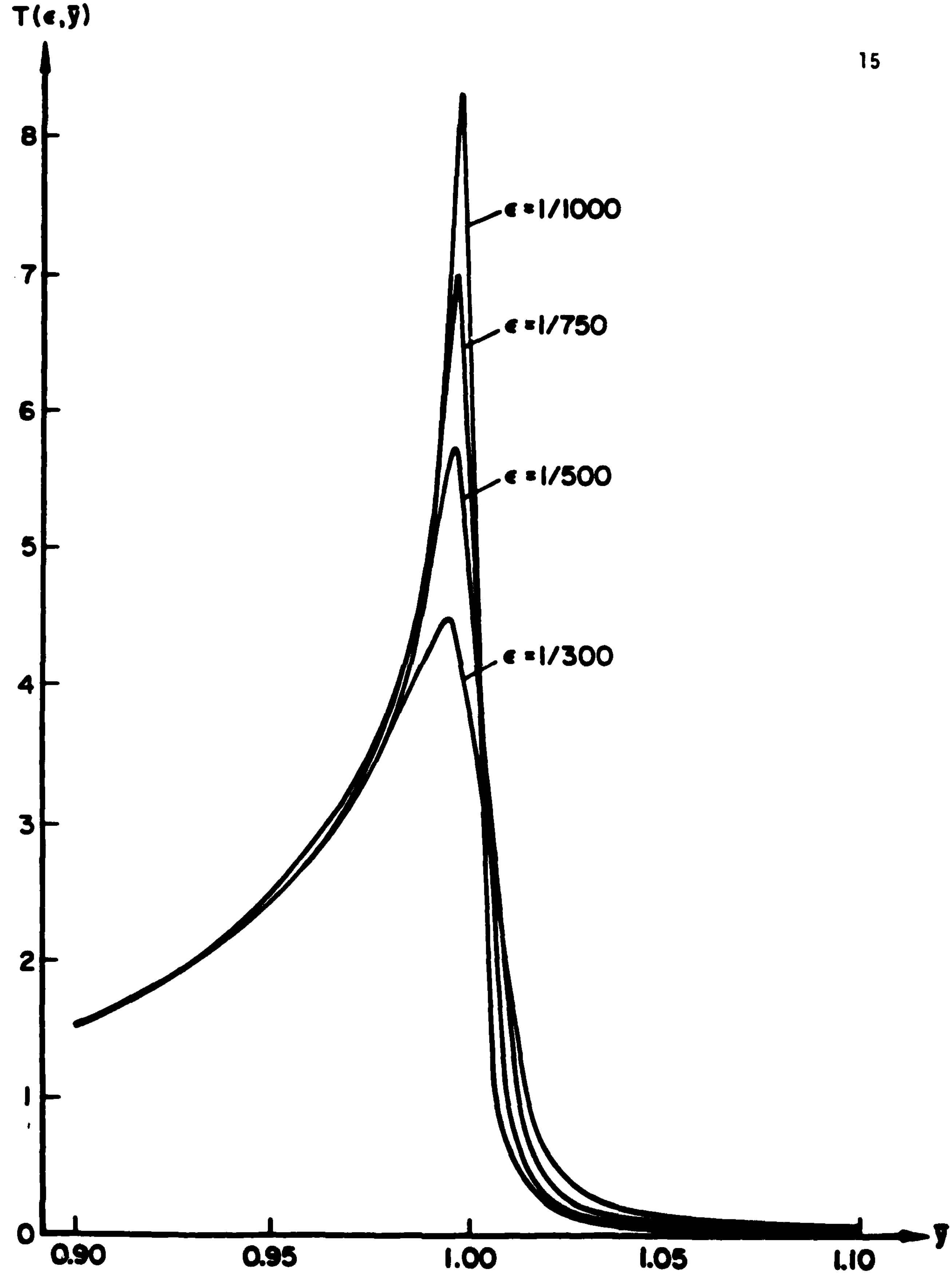

Figure 3: Nondimensional Normal Stress at the Surface [6]

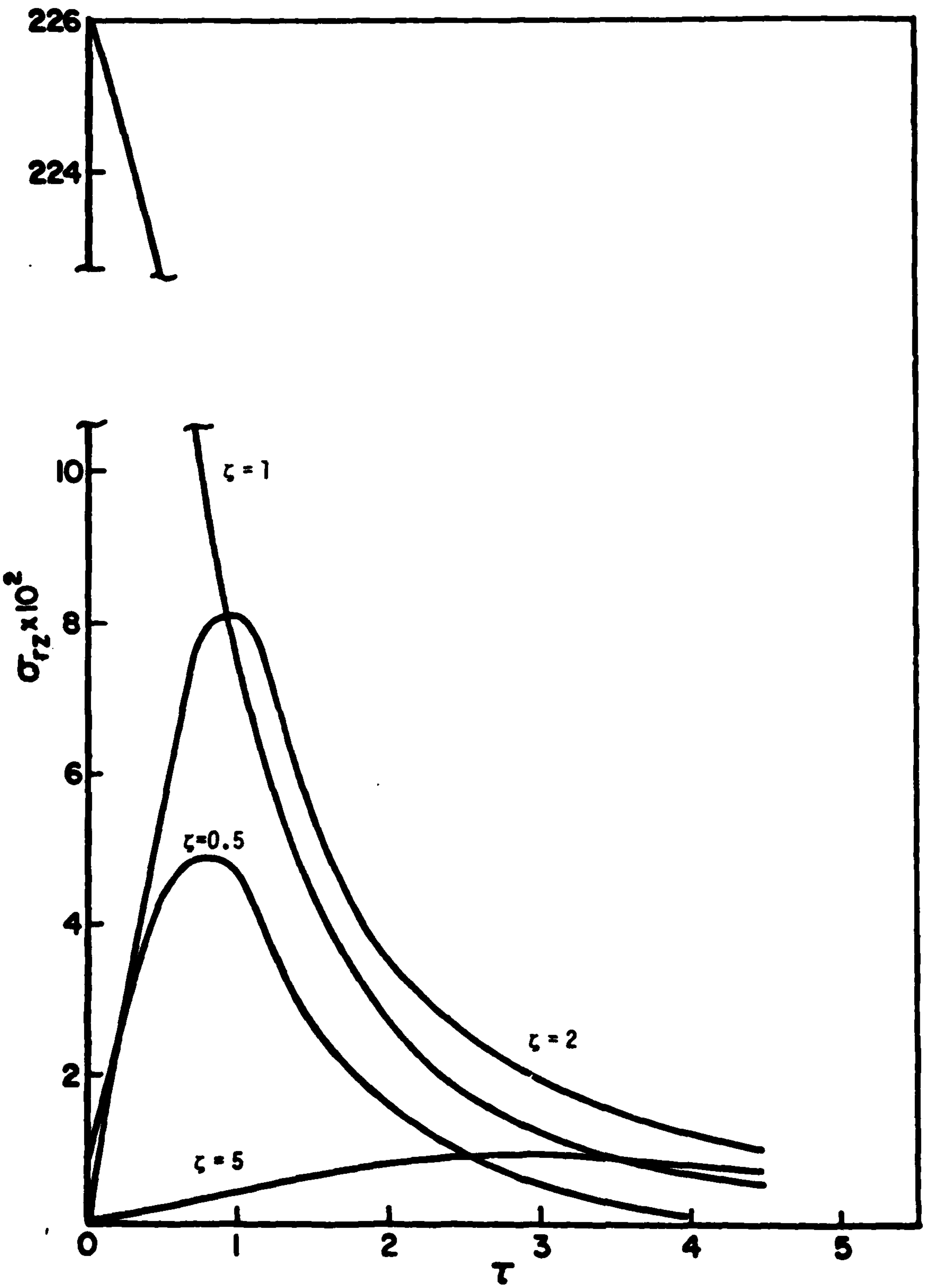

Figure 4: Nondimensional Shear Stress vs. Time [8]

$\tau = \frac{\mu t}{\rho R^2} \qquad \sigma_{tz} = -\frac{R\, t_{rz}}{\mu\, V_0} \qquad \zeta = r/R$

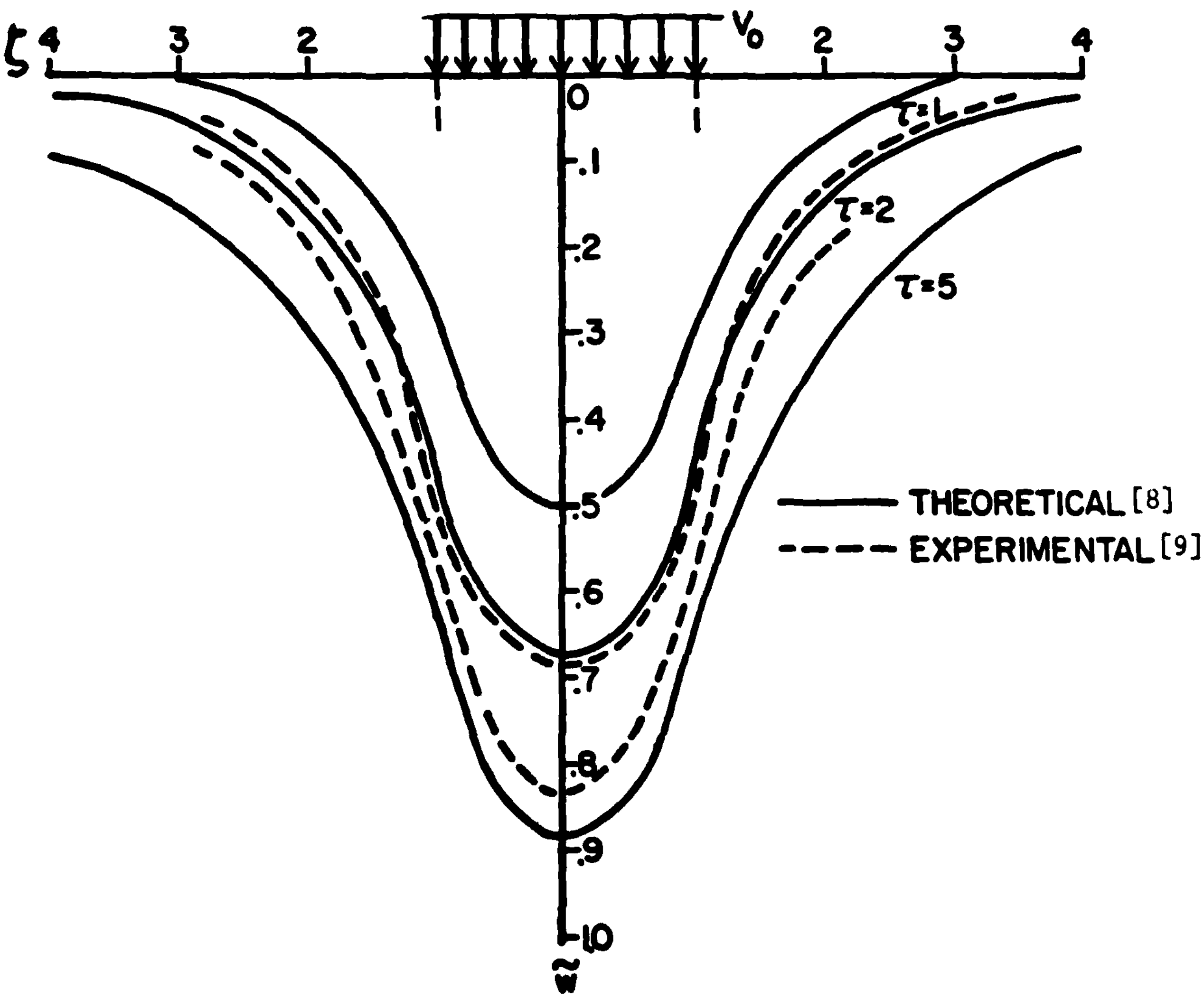

Figure 5: Nondimensional Displacement vs. ζ [8]

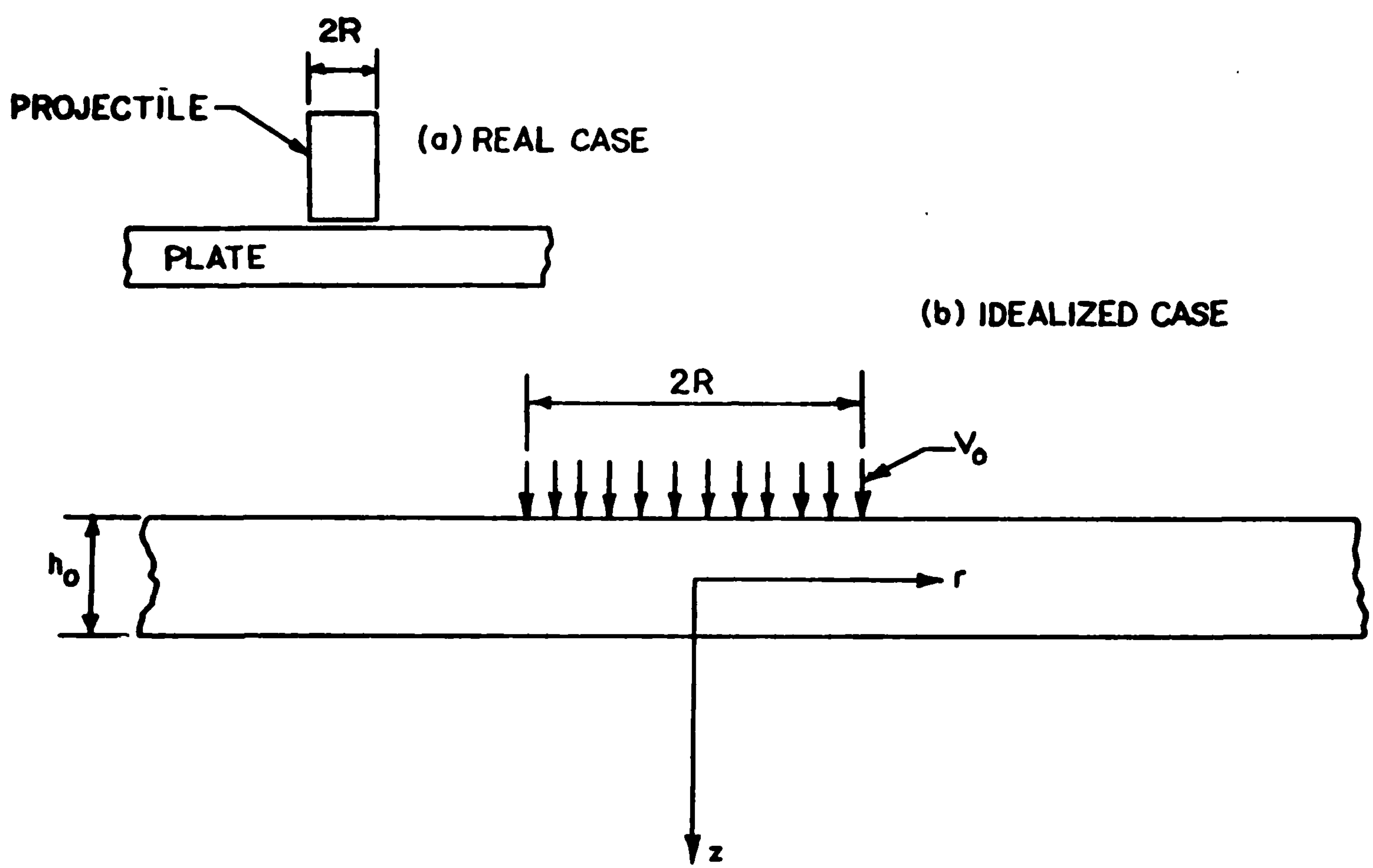

Figure 6: Perforation of a Visco-Plastic Plate [10]

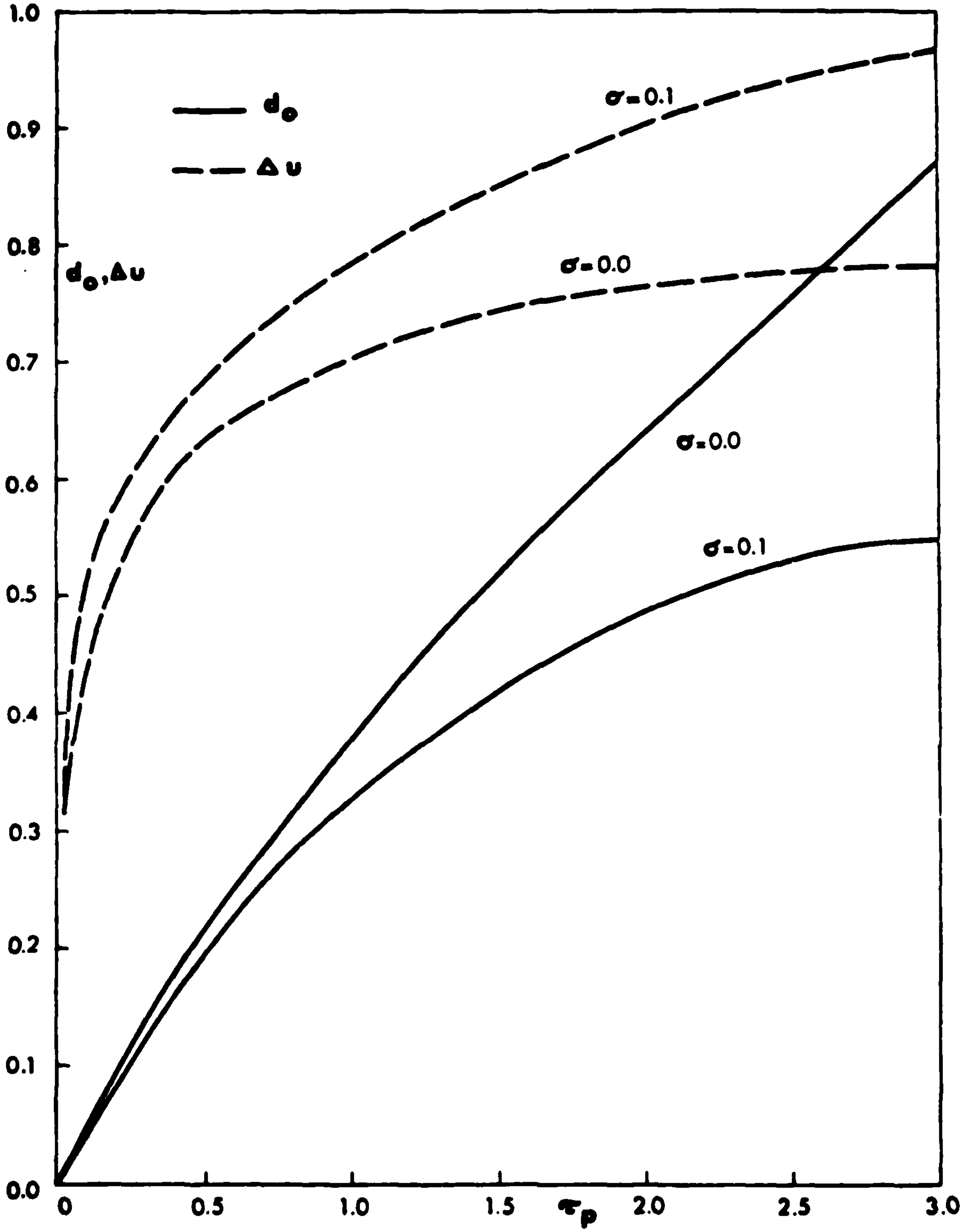

Figure 7: Variations of Plate Thickness and Velocity Drop with Perforation Time [10]

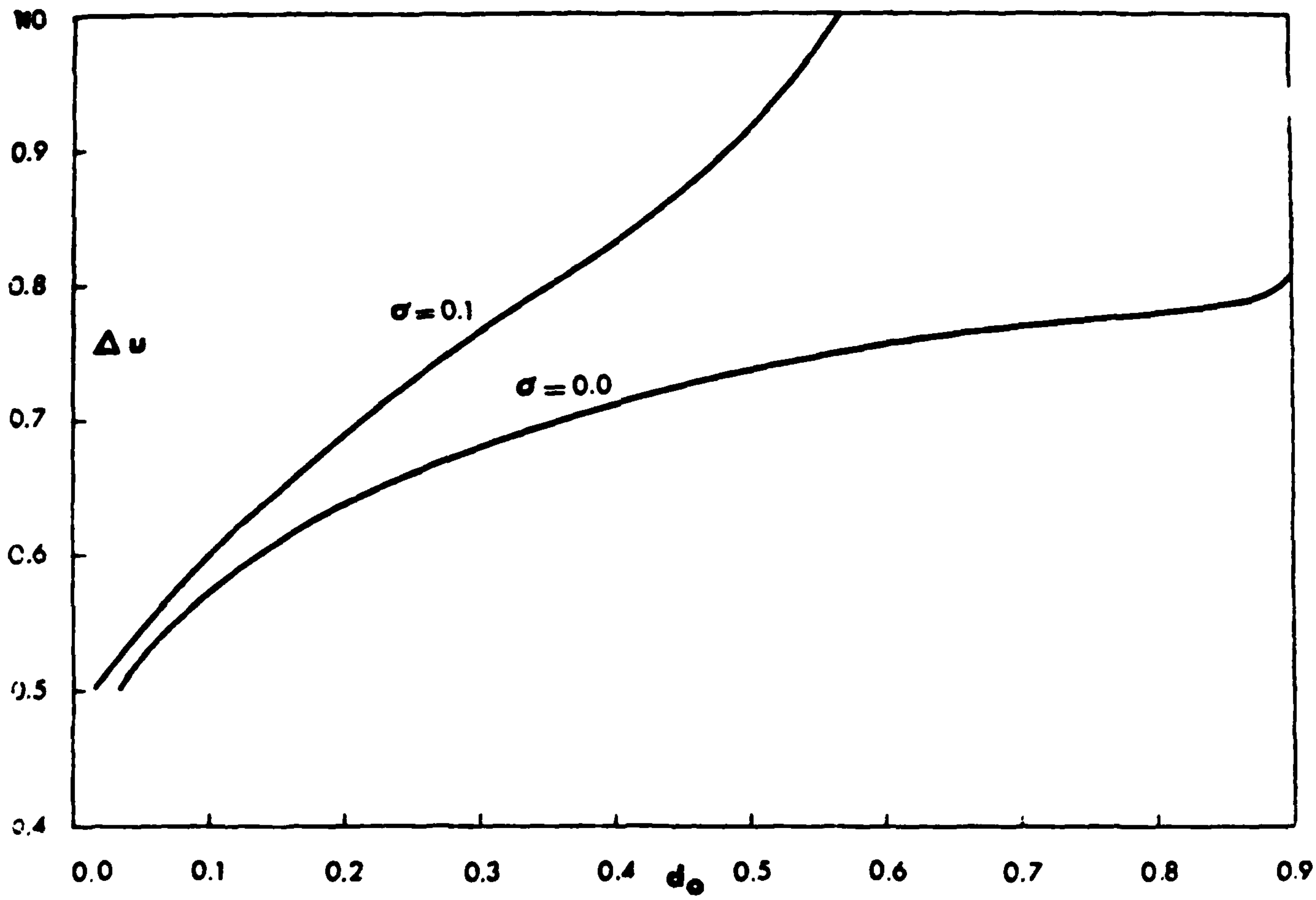

Figure 8: Velocity Drop Changes with Plate Thickness [10]

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VI. LIST OF ARO-D SPONSORED PUBLICATIONS AND TECHNICAL REPORTS

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2. "Crack-Tip Problem in Nonlocal Elasticity," (with C.G. Speziale and B.S. Kim), J. Mech. Phys. Solids, 1977, Vol. 25, pp. 339-355.

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8. "On Nonlocal Plasticity," 81-SM-4.

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